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ELECTROMAGNETIC COUPLING DUE TO APERTURES IN MISSILE BODIES

TECHNICAL REPORT

NO. 5455

CONTRACT N178-7604

Prepared For

THE U.S. NAVAL WEAPONS LABORATORY

DAHLGREN, VA.

by

Jansky & Bailey

A DIVISION OF ATLANTIC RESEARCH CORP.

ALEXANDRIA, VA.

XEROX

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CLIFTON MERCHANT

Approved By:

W.T.Dickinson

W.T.Dickinson

Prepared By:

Clifton Merchant

Clifton Merchant

Jansky & Bailey

A DIVISION OF ATLANTIC RESEARCH CORP.

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ABSTRACT

This report considers the problem of electromagnetic coupling between the interior and exterior of cylindrical missile bodies due to apertures in the missile body. Expressions are derived which can be used to determine the electromagnetic field at any point (interior and exterior) in terms of a field incident to the missile.

SECTION I

1.1 Introduction

It is possible for microwave energy to enter the cavity formed by the missile body through an opening or aperture in the missile body. The microwave energy may then induce a current in the squib and its wiring and cause a hazardous condition. Therefore, in order to evaluate the HERO problem at microwave frequencies, it is necessary to relate the field incident to the missile to the current induced in the squib.

It is the purpose of this work to relate the field incident to the missile to the field on the inside of the missile in terms of the aperture of the missile body. To simplify the problem, the missile shall be considered as an infinitely long cylinder with a radius, a. In a later work, the missile shall be treated as a closed cavity. The walls of the cylinder shall be considered to be perfectly conducting and have an infinitesimal thickness. The cylinder has an aperture of rectangular shape which is bounded axially by the planes $Z = Z_1$ and $Z = -Z_1$, and bounded in azimuth by the planes $\phi = \phi_1$ and $\phi = \phi_2$. The cylinder, aperture, and coordinate system are shown in Fig. 1-1; the aperture is denoted by τ and the conducting surface as σ.

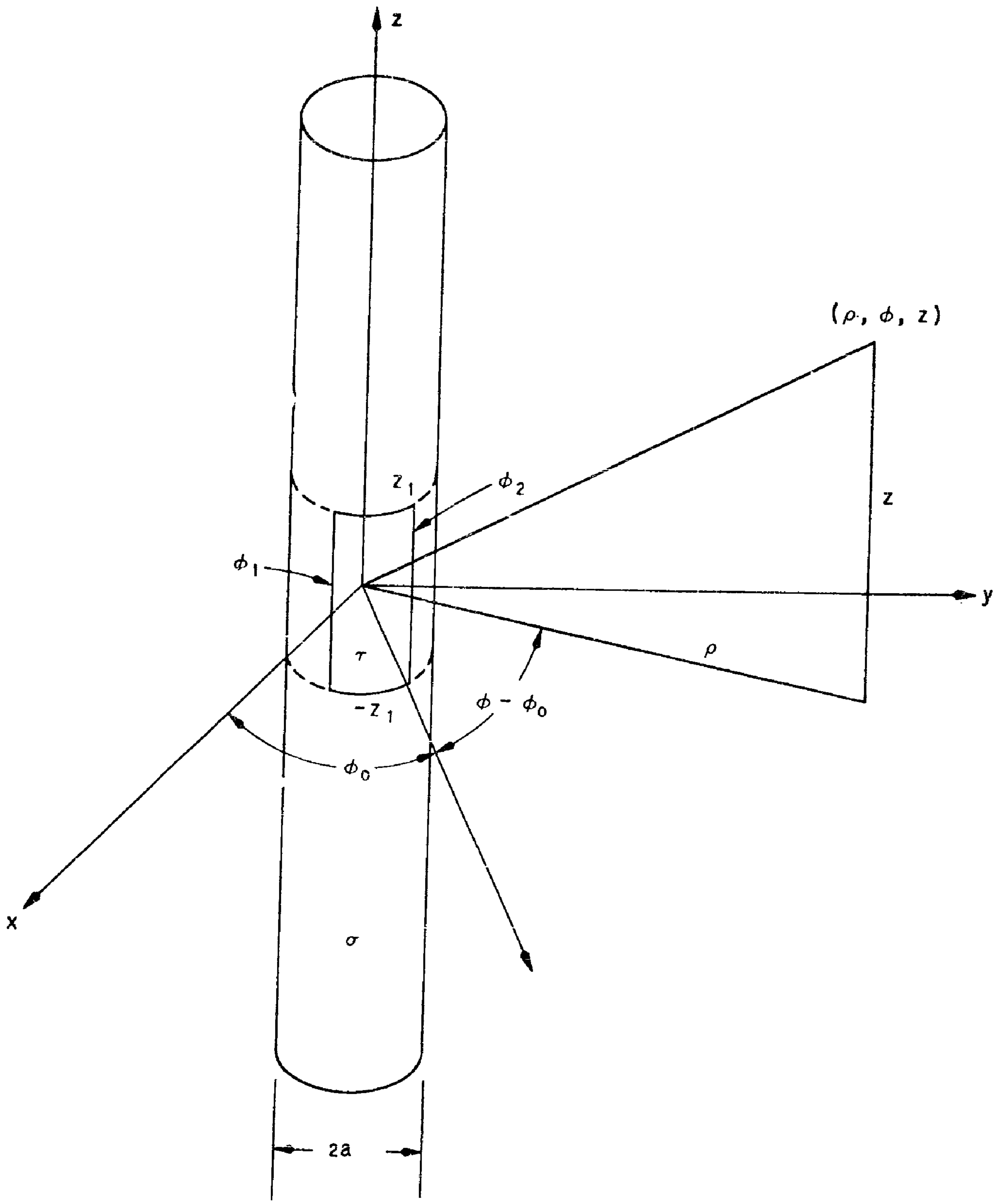

FIG. 1-1
CONDUCTING CYLINDER WITH APERTURE

SECTION II

2.1 Theory of Diffraction

Finding the field inside the cylinder in terms of the field incident on the cylinder is a problem in diffraction. A mathematically exact solution of the diffraction problem should satisfy the following requirements[1]:

a. The total field must be a solution of Maxwell's equations.

b. On the surface of the conducting cylinder, σ, the condition of field continuity requires that

$$\bar{n} \times \bar{E} = 0 \tag{1a}$$

$$\bar{n} \cdot \bar{H} = 0 \tag{1b}$$

where $\bar{n}$ is a unit vector, normal to the cylinder and directed into the cylinder.

c. All the components of the electromagnetic field must be continuous in crossing the aperture from the space $\rho > a$ to the space $\rho < a$.

The electromagnetic field, $\bar{E}$ and $\bar{H}$, at any point in space, is split up into an incident field, $\bar{E}_I$ and $\bar{H}_I$, a reflected field, $\bar{E}_r$ and $\bar{H}_r$, associated with the reflected wave that exists when the cylinder is complete, and a perturbed field, $\bar{E}_p$ and $\bar{H}_p$, due

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[1] Bekefi, G., Diffraction of Electromagnetic Waves by an Aperture in an Infinite Screen, Eaton Electronics Research Lab., McGill University, Montreal, Canada, TR No. 22, Contract No. AF 19(122)-81; September 1952.

to the aperture alone. The field in the space $\rho > a$ is:

$$\bar{E}_1 = \bar{E}_i + \bar{E}_r + \bar{E}_{p1} \tag{2a}$$

$$\bar{H}_1 = \bar{H}_i + \bar{H}_r + \bar{H}_{p1} \tag{2b}$$

and the field in the space $\rho < a$ is:

$$\bar{E}_2 = \bar{E}_{p2} \tag{3a}$$

$$\bar{H}_2 = \bar{H}_{p2} \tag{3b}$$

The subscripts 1 and 2 denote the space $\rho > a$ and $\rho < a$ respectively. The continuity conditions in the aperture, τ, are fulfilled if[2]:

$$\bar{n} \cdot \bar{E}_{p2} = -\bar{n} \cdot \bar{E}_{p1} \tag{4a}$$

$$\bar{n} \times \bar{H}_{p2} = -\bar{n} \times \bar{H}_{p2} \tag{4b}$$

As a result of these relationships, it can be shown that within the aperture the components of $\bar{H}$ parallel to the cylinder and the components of $\bar{E}$ normal to the cylinder are to be identified with the respective components of the incident radiation.

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[2]Silver, S., University of California, Antenna Laboratory, Berkely, California, TR No. 163; 1949.

SECTION III

3.1 The Perturbed Field

The perturbed field is that portion of the total field due to the aperture. To find the perturbed field, the tangential electric field in the opening of the cylinder is assumed to be

$$E_\phi\ (a, \phi, Z) = f_1\ (\phi, Z) = F_1\ (\phi)\ G_1\ (Z) \tag{5a}$$

$$E_Z\ (a, \phi, Z) = f_2\ (\phi, Z) = F_2\ (\phi)\ G_2\ (Z) \tag{5b}$$

An expression can now be found for the Hertzian vector of the magnetic type, $\Pi_p{}^*$, for the perturbed field. The inhomogeneous wave equation is[3]:

$$\nabla^2\ \bar{\Pi}_p{}^* + k^2\ \bar{\Pi}_p^* = -\bar{M} \tag{6}$$

where

$k = \omega \sqrt{\mu\ \epsilon}$

$M = \frac{1}{j\ \omega}\ (\bar{n} \times \bar{E})$ = density of the distribution of magnetic moment.

Dividing Eq. (6) into vector components gives three partial differential equations for the vector components of the Hertzian vector in terms of the electric field in the opening. The equations are:

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[3]Stratton, J.A., Electromagnetic Theory, McGraw-Hill Book Co., Inc., New York, N.Y., p. 30; 1941.

$$\frac{\partial^2 \Pi_p{}^*{}_\rho}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \Pi_p{}^*{}_\rho}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \Pi_p{}^*{}_\rho}{\partial \phi^2} + \frac{\partial^2 \Pi_p{}^*{}_\rho}{\partial Z^2}$$

$$+ \left(k^2 - \frac{1}{\rho^2}\right) \Pi_p{}^*{}_\rho - \frac{2}{\rho^2} \frac{\partial \Pi_p{}^*{}_\phi}{\partial \phi} = 0 \tag{7a}$$

$$\frac{\partial^2 \Pi_p{}^*{}_\phi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \Pi_p{}^*{}_\phi}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \Pi_p{}^*{}_\phi}{\partial \phi^2} + \frac{\partial^2 \Pi_p{}^*{}_\phi}{\partial Z^2}$$

$$+ \left(k^2 - \frac{1}{\rho^2}\right) \Pi_p{}^*{}_\phi + \frac{2}{\rho^2} \frac{\partial \Pi_p{}^*{}_\rho}{\partial \phi} = - \frac{F_2(\phi)\ G_2\ (Z)}{j\ \omega}\ \delta\,(\rho - a) \tag{7b}$$

$$\frac{\partial^2 \Pi_p{}^*{}_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \Pi_p{}^*{}_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \Pi_p{}^*{}_z}{\partial \phi^2} + \frac{\partial^2 \Pi_p{}^*{}_z}{\partial Z^2}$$

$$+ k^2 \Pi_p{}^*{}_z = \frac{F_1\ (\phi)\ G_2\ (Z)}{j\ \omega}\ \delta\ (\rho - a) \tag{7c}$$

The components of $\overline{\Pi}^*{}_p$ which satisfy Eqs. (7) must satisfy the boundary conditions that the tangential electric field vanishes on the conducting surface, and is equal to the electric field in the opening. That is

$$\lim_{\rho \to a} \left[- j\omega\mu \left(\frac{\partial \Pi_p{}^*{}_\phi}{\partial \rho} + \frac{\Pi_p{}^*{}_\phi}{\rho} - \frac{1}{\rho} \frac{\partial \Pi_p{}^*{}_\rho}{\partial \phi} \right) \right] = \begin{cases} 0 & \text{on } \sigma \\ E_z\ (a,\ \phi,\ Z) & \text{on } \tau \end{cases} \tag{8a}$$

$$\lim_{\rho \to a} \left[- j\omega\mu \left(\frac{\partial \Pi_p{}^*{}_\rho}{\partial Z} - \frac{\partial \Pi_p{}^*{}_z}{\partial \rho} \right) \right] = \begin{cases} 0 & \text{on } \sigma \\ E_\phi \; (a, \phi, Z) & \text{on } \tau \end{cases} \tag{8b}$$

Once the Hertzian vector has been found, the electric and magnetic field components of the perturbed field can be found from

$$\bar{E}_p = -j \omega \mu \; (\nabla \times \bar{\Pi}_p{}^*) \tag{9a}$$

$$\bar{H}_p = \nabla \times \nabla \times \bar{\Pi}_p{}^* \tag{9b}$$

SECTION IV

4.1 Diffraction by an Aperture in a Hollow Cylinder

Equations will now be found which can be used to solve the diffraction problem. Two cases will be considered. The first case is polarization of the incident wave parallel to the axis of the cylinder; the second case is polarization transverse to the axis of the cylinder.

4.1.1 Incident Polarization Parallel to Cylinder

The incident electromagnetic wave traveling in the positive x direction is

$$\bar{E}_I = \bar{k} \, E_0 \, e^{-jkx} \tag{10}$$

with the harmonic time variation, $e^{-j\omega t}$, understood. Expanding into cylindrical coordinates gives

$$\bar{E}_I = \bar{k} \, E_0 \sum_{n=0}^{\infty} \epsilon_n \, (-j)^n \cos n\phi \, J_n \, (k\rho) \tag{11}$$

where

$\epsilon_0 = 1, \quad \epsilon_n = 2 \quad (n \neq 0)$

$J_n \, (k\rho)$ is the Bessel function of the first kind, order n.

The reflected wave is given by a similar series,

$$\bar{E}_r = -\bar{k}\ E_o \sum_{n=0}^{\infty} \epsilon_n \ (-j)^n \cos n\phi \ \frac{J_n\ (ka)}{H_n^{(2)}\ (ka)}\ H_n^{(2)}\ (k\rho) \qquad (12)$$

where $H_n^{(2)}\ (k\rho)$ is the Hankel function of the second kind.

The magnetic field components of the incident and reflected fields can be found from $\bar{H} = -\ \frac{1}{j\omega\mu}\ \nabla \times \bar{E}$

$$\bar{H}_i + \bar{H}_r = \frac{E_o}{jk\eta} \sum_{n=0}^{\infty} \epsilon_n \ (-j)^n \left\{ \bar{I}_\rho \ \frac{n}{\rho} \sin n\phi \left[J_n\ (k\rho) \right.\right.$$

$$\left. - \frac{J_n\ (ka)}{H_n^{(2)}(ka)}\ H_n^{(2)}\ (k\rho) \right] + \bar{I}_\phi\, k \ \cos n\phi \left[J_n'\ (k\rho) \right.$$

$$\left.\left. - \frac{J_n\ (ka)}{H_n^{(2)}(ka)}\ H_n^{(2)'}\ (k\rho) \right] \right\} \qquad (13)$$

The boundary conditions require that the tangential magnetic components of the electromagnetic field be continuous across the opening τ. This requires that

$$\lim_{\rho \to a} H_{z1} = \lim_{\rho \to a} H_{z2} \qquad (14a)$$

$$\lim_{\rho \to a} H_{\phi 1} = \lim_{\rho \to a} H_{\phi 2} \qquad (14b)$$

From Eqs. (2), (3), (4) and (13), the relationships in Eq. (14) become

$$\lim_{\rho \to a} \left\{ \frac{1}{\rho^2}\left(-\frac{\partial^2 \Pi_p{}^* z}{\partial \phi^2}\right) + \frac{1}{\rho}\left(\frac{\partial \Pi_p{}^* \rho}{\partial Z} - \frac{\partial \Pi_p{}^* z}{\partial \rho} + \frac{\partial^2 \Pi_p{}^* \phi}{\partial \phi\, \partial Z}\right) \right.$$

$$\left. + \frac{\partial^2 \Pi_p{}^* \rho}{\partial \rho\, \partial Z} - \frac{\partial^2 \Pi_p{}^* z}{\partial \rho^2} \right\} = 0 \qquad (15a)$$

$$\frac{E_o}{k a \eta \Pi} \sum_{n=o}^{\infty} \epsilon_n (-j)^n \frac{\cos n \phi}{H_n{}^{(2)}(k a)}$$

$$= \lim_{\rho \to a} \left\{ \frac{1}{\rho^2}\left(\Pi_p{}^* \phi - \frac{\partial \Pi_p{}^* \rho}{\partial \phi}\right) + \frac{1}{\rho}\left(\frac{\partial^2 \Pi_p{}^* z}{\partial Z\, \partial \phi}\right.\right.$$

$$\left.\left. - \frac{\partial \Pi_p{}^* \phi}{\partial \rho} + \frac{\partial^2 \Pi_p{}^* \rho}{\partial \rho\, \partial \phi}\right) - \frac{\partial^2 \Pi_p{}^* \phi}{\partial Z^2} - \frac{\partial^2 \Pi_p{}^* \phi}{\partial \rho^2} \right\} \qquad (15b)$$

Equations (15) together with Eqs. (7) form a system of equations from which the Hertzian vector of the perturbed field and the field in the aperture may be found.

4.1.2 Incident Polarization Transverse to Cylinder

The incident wave traveling in the positive x direction is

$$\bar{E}_I = -j\, E_o\, e^{-j k x} \qquad (16)$$

If the same analysis is followed as in the previous case, the equations for continuity of the tangential magnetic field across the opening become

$$\lim_{\rho \to a} \left\{ \frac{1}{\rho^2} \left(\Pi_p{}^*{}_\phi - \frac{\partial \Pi_p{}^*{}_\rho}{\partial \phi} \right) + \frac{1}{\rho} \left(\frac{\partial^2 \Pi_p{}^*{}_z}{\partial Z \, \partial \phi} - \frac{\partial \Pi_p{}^*{}_\phi}{\partial \rho} + \frac{\partial^2 \Pi_p{}^*{}_\rho}{\partial \rho \, \partial \phi} \right) - \frac{\partial^2 \Pi_p{}^*{}_\phi}{\partial Z^2} - \frac{\partial^2 \Pi_p{}^*{}_\phi}{\partial \rho^2} \right\} = 0 \qquad (17a)$$

$$\frac{j E_o}{k a \eta \Pi} \sum_{n=o}^{\infty} \epsilon_n (-j)^n \frac{\cos n\phi}{H_n^{(2)\prime}(ka)}$$

$$= \lim_{\rho \to a} \left\{ \frac{1}{\rho^2} \left(- \frac{\partial^2 \Pi_p{}^*{}_z}{\partial \phi^2} \right) + \left(\frac{1}{\rho} \quad \frac{\partial \Pi_p{}^*{}_\rho}{\rho \, Z} - \frac{\partial \Pi_p{}^*{}_z}{\partial \rho} + \frac{\partial^2 \Pi_p{}^*{}_\phi}{\partial \phi \, \partial Z} \right) + \frac{\partial^2 \Pi_p{}^*{}_\rho}{\partial \rho \, \partial Z} - \frac{\partial^2 \Pi^*{}_z}{\partial \rho^2} \right\} \qquad (17b)$$

Equations (17), together with Eqs. (7), form a system of equations from which the Hertzian vector of the perturbed field and the field in the aperture may be found.

SECTION V

5.1 Application of Equations

It is possible to apply the equations developed in this work to find the electromagnetic field inside a circular cylinder in terms of the field incident to the cylinder. The equations can also be applied to find the total field external to the missile, i.e., the reflected and perturbed field in addition to the incident field.

Further effort will be carried out to extend this method to cylinders with finite lengths and closed interiors.

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